การโปรแกรมเชิงเส้นแบบพอสสิบิลิสติกสำหรับการวางแผนการผลิตรวมที่มีการทดแทนแรงงานในสภาพแวดล้อมแบบเครื่องจักรวางขนาน

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หน่วยวิจัยทางด้านการวิจัยดำเนินงานและสถิติอุตสาหกรรม ภาควิชาวิศวกรรมอุตสาหการ คณะวิศวกรรมศาสตร์
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บทคัดย่อ

งานวิจัยนี้นำเสนอวิธีการโปรแกรมเชิงเส้นแบบพอสสิบิลิสติก สำหรับแก้ปัญหาการวางแผนการผลิตรวมที่มีหลายผลิตภัณฑ์ ซึ่งมีการพิจารณาทดแทนแรงงานในสภาพแวดล้อมแบบเครื่องจักรวางขนานกัน โดยที่ค่าพยากรณ์ความต้องการสินค้า ต้นทุนวัตถุดิบ และต้นทุนอุปกรณ์มีความคลุมเครือ วิธีที่นำเสนอพยายามหาคำตอบที่มีค่าผลกำไรสูงสุด โดยใช้กลยุทธ์ในการพิจารณาไปพร้อมๆ กันของ การหาค่ามากที่สุดของคำตอบที่มีโอกาสเกิดขึ้นได้มากสุด คำตอบในมุมมองแง่ร้ายและคำตอบในมุมมองแง่ดีของผลกำไรที่มีความคลุมเครือ การทดแทนแรงงานด้วยอุปกรณ์ถูกนำมาพิจารณาในตัวแบบเพื่อเพิ่มความสามารถและเพิ่มประสิทธิภาพของระบบ ตัวแบบที่นำเสนอสามารถหาคำตอบที่เป็นที่ยอมรับพึงพอใจมากกว่า และมีข้อมูลมากขึ้นกว่าการหาคำตอบด้วยวิธีดั้งเดิม นอกจากนี้ยังสามารถควบคุมเพื่อให้ได้แผนการผลิตที่ต้องการตามความชอบของผู้ตัดสินใจได้ นอกจากนี้กรณีศึกษาจริงได้ถูกนำมาแสดงไว้ด้วย

คำสำคัญ: วิธีการโปรแกรมเชิงเส้นแบบพอสสิบิลิสติก, การวางแผนการผลิตรวม, การทดแทนแรงงาน, เครื่องจักรวางขนาน, การโปรแกรมแบบฟัซซี่, ข้อมูลที่คลุมเครือ

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Possibilistic Linear Programming for Aggregate Production Planning with Labor Replacement in a Parallel Machine Environment

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Abstract

This work presents a Possibilistic Linear Programming (PLP) approach for solving a multi-product. Aggregate Production Planning (APP) problem with labor replacement in a parallel machine environment where forecasted demand, material cost, and equipment cost is imprecise. The proposed approach attempts to maximize the total profit. It uses the strategy of simultaneously maximizing the most possible, pessimistic, and optimistic values of the imprecise total profit. Labor replacement by equipment is considered in the model for capacity expansion and increasing system's efficiency. The proposed model yields an efficient compromised solution, which is more preferable and contains more information than conventional approaches. It can also be easily manipulated to obtain the preferred plan according to a decision maker's preference. A real industrial case is also demonstrated.

Keywords: Possibilistic Linear Programming; Aggregate Production Planning; Labor Replacement; Parallel Machines; Fuzzy Programming; Imprecise Information

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1. Introduction

Aggregate Production Planning (APP) is a medium-term planning whereby its planning horizon is usually from 6 to 18 months [1]. It links operations with strategies and plays a key role in enterprise resource planning and organizational integration [2]. Since mid 1950's researchers have developed numerous models to solve the APP problem [3,4]. There are a number of approaches to APP: mathematically optimal procedures [3-6], simulation and search methods [7-8], and heuristics [9-10]. Among the optimal models, Linear Programming (LP) and Goal Programming (GP) have received the widest acceptance [11].

Although there are a number of APP models, most of them assume that the production capacity can be varied by managing workforce level under fixed and limited capacity of hardware. In practice workforce level can be replaced by equipment such as robots, material handlings, or automated systems, which can also enhance the system efficiency. Competitiveness and profitability can also be increased. The ultimate challenge of technologies pushes companies to a competitive environment. This kind of replacement has not been mentioned in the literature of APP yet. Enterprises face technology replacement decisions more frequently as technology upgrades accelerate [12]. Capacity decisions affect product lead times, customer responsiveness, operating costs, and a firm's ability to compete. Inadequate capacity can lost customers and limit growth. Excess capacity can drain a company's resources and prevent investments in more lucrative ventures. When, how much, and in what form to alter capacity are critical decisions [13]. Most of literature of replacement focus on equipment replacement rather than labor replacement (by equipment) [14-16]. However, labor replacement is also critical for practical consideration. Even though investment may lead to high cost but for medium to long term it may increase the total profit of the company. Moreover, most APP models assume that all required input data can be uniquely determined. These models might also have some practical difficulties since a Decision Maker (DM) frequently has insufficient or unobtainable information on how to specify crisp information. These data are typically uncertain in nature. In dealing with uncertainty, stochastic and fuzzy models are used. Stochastic models can deal with uncertainty but they are hard to solve and statistical estimations prove inefficient because of the lack of statistical observations [17]. Moreover, it might not give us the right meaning to solve some practical decision-making problems [18]. Other forms of uncertainties are based on fuzzy set theory. Fuzzy set theory provides better tools to represent a problem with fuzzy/imprecise input data. Zimmermann [19] first introduced fuzzy set theory into the conventional LP problem in 1976. Since then, fuzzy mathematical programming has developed into several fuzzy optimization methods for solving APP problems [20-25]. Another main approach in dealing with fuzzy models is the possibility theory, which was introduced by Dubois and Prade in 1983 [26]. A grade of possibility is used to indicate the subjective or objective degree of the occurrence of an event [27]. Lai and Hwang [18] developed an auxiliary multiple objective linear programming (MOLP) model for solving a Possibilistic Linear Programming (PLP) problem with imprecise objective and/or constraint coefficients. Hsu and Wang [28] and Wang and Liang [29] applied Lai and Hwang's PLP approach to manage decision problems. However, interactive changing values of the degree of satisfaction of relative measure objective functions are difficult.

Therefore, the aim of this study is to develop a PLP approach for solving the APP problem with labor replacement where forecasted demand, material cost, and equipment cost are imprecise. The proposed model is suitable for a parallel machine environment which has similar production lines or machines. This kind of shop may need to install a number of equipment for increasing the production capacity instead of determination of increasing capacity from labor as in conventional APP models. Examples of this kind of shop are plastic injection shops or assembly shops. A real case study of plastic injection factory is provided. This company was considering replacement of workers by robot arms for shortening the loading and unloading times, improving system efficiency and competitiveness. The proposed model was applied and found suitably to provide the production plan.

2. Problem Formulation

2.1 Problem Description and Notations

Traditional factories try to shift their factories to automated factories for increasing their efficiency and capacity expansion. Especially, in lean production elementary step is to increase flexibility of the system. Workers are replaced by equipment to increase technology advancement, plant modernization, product quality, flexibility, competitive position, and survival of the company. A parallel machine environment is considered in the proposed APP model. There are three types of resources for production; labor, equipment, and machine. Equipment in this context means robot arms, material handlings, or automated systems. Equipment can replace labor to shorten the loading and unloading times of work-pieces and increase system's capacity. The equipment also has shorter processing time than the labor. Thus, when the labor is replaced by the equipment, the available productive time of the machine is increased because the non-productive time is shorter. The output is increased because of higher productive time and shorter processing time. However, investment is needed.

In APP problems, the DM must determine the best decisions for each product (*i*) of all *N* products according to the adjustable capacity for meeting production requirements for each time period (*t*) of the planning horizon (*T*). In parallel machine shop most workers work in shift (*f*). It is assumed that there are 3 shifts per day.

Notation of variables:

$P_{i,t}$: regular time production quantity of product *i* in period *t* (units);

$P^w_{i,t}, P^e_{i,t}$: regular time production quantity of product *i* in period *t* by worker and by equipment (units);

$O_{i,t}$: overtime production quantity of product *i* in period *t* (units);

$O^w_{i,t}, O^e_{i,t}$: overtime production quantity of product *i* in period *t* by worker and by equipment (units);

$S_{i,t}$: subcontracting production quantity of product *i* in period *t* (units);

$I_{i,t}$: inventory quantity of product *i* at the beginning of period *t* (units);

$W_{f,t}$: total workforce level for shift *f* in period *t* (workers);

H_t, L_t: hired and laid-off workforce at the beginning of period *t* (workers);

$B_{i,t}$: backorder quantity of product *i* at the beginning of period *t* (units);

E_t : cumulative number of equipment in period *t* (units);

A_t : additional equipment at the beginning of period *t* (units);

M_t: available machine or system capacity in period *t* (machine-hours);

Notation of parameters and constants:

r_i: price of product *i* per unit (currency unit/unit);

$\tilde{c}_{mi}$: imprecise material cost of product *i* per unit (currency unit/unit);

c_{OTi}: overtime labor cost for product *i* (currency unit/unit);

c_{Ii}, c_{Bi}: inventory holding cost and backordering cost of product *i* per period per unit (currency unit/unit);

c_{Si}: subcontracting cost of product *i* per unit (currency unit/unit);

c_{Wt}: regular time wages in period *t* ((currency unit/man);

c_{Ht}, c_{Lt}: hiring cost, and firing cost in period *t* (currency unit/man);

$\tilde{c}_E$: imprecise equipment cost per period (converted from capital recovery and operating cost with time value of money in concern) (currency unit/unit);

a_i : labor time needed for product *i* per unit (man-hour/unit);

b_i : equipment time needed for product i per unit (hour/unit);

c_i : machine time needed for product *i* per unit (machine-hour/unit);

δ : regular time per shift (hour);

γ: increased number of machine-hour per unit of equipment in each period;

n_t : normal working day in each period *t* (days);

h_t : holiday that can assign overtime in period *t* (days);

$W_{t\max}$: maximum available workforce in period *t* (man);

$W_{t\min}$: minimum available workforce in period *t* (man);

M_t: available machine or system capacity in period *t* (machine-hour);

$MC_{t\max}$: maximum possible available machine or system capacity in period *t* if equipment are additionally invested (machine-hour);

$E_{\max}$: maximum number of equipment (units);

$\tilde{F}_{t\max}$: imprecise available financial resource in period *t* (currency unit);

M_0: initial available machine capacity (machine-hour);

E_0 : initial number of equipment (units).

In this study, the material cost, $\tilde{c}_{mi}$ and equipment cost, $\tilde{c}_E$ are considered as imprecise information because these two costs are set by outsiders that are difficult to control and estimate. Material cost may increase or decrease according to currency rate or economic situation. Estimated value of equipment and it salvage value which is considered as annual worth may be different from actual value. So imprecision of these information are needed. Moreover, financial resource for each period, $\tilde{F}_{t\max}$ is also imprecise because it is an estimated fund that may change according to interest rate, currency rate, or economic conditions. So, possibilistic distributions are used to represent these subjective and objective degrees of the occurrence of these events.

2.2 Possibilistic linear programming for aggregate production planning

2.2.1 Objective functions

Conventionally, revenue, cost or profit function is selected to be the objective function of APP problems. Among these objective functions, the profit function is the most preferable. Profit equals to total sales minus total cost. Total sales revenue is based on total demands and lost sales at the end of the planning horizon. Total cost composes of material cost, overtime cost, inventory cost, backordering cost, subcontracting cost, labor cost and equipment cost. In this research the imprecise objective function ($\tilde{Z}$) can be illustrated as:

$$\tilde{Z} = \sum_{i=1}^{N}\sum_{t=1}^{T} r_i \tilde{D}_{i,t} - \sum_{i=1}^{N} r_i B_{i,T} - \sum_{i=1}^{N}\sum_{t=1}^{T} \tilde{c}_{mi}\left(P_{i,t} + O_{i,t} + S_{i,t}\right) -$$

$$[\sum_{i=1}^{N}\sum_{t=1}^{T}\left(c_{OTi} a_i O^{w}_{i,t} + c_{Ii} I_{i,t} + c_{Bi} B_{i,t} + c_{Si} S_{i,t}\right) +$$

$$\sum_{t=1}^{T}\left(\left(\sum_{f=1}^{3} c_{Wt} W_{f,t} + c_{Ht} H_t + c_{Lt} L_t\right) + \tilde{c}_E E_t\right)], \quad (1)$$

where $\tilde{D}_{i,t}$ denotes the imprecise forecast demand of product *i* in period *t*. $\tilde{c}_{mi}$ and $\tilde{c}_E$ represent imprecise material cost of product *i* and capital recovery cost and operating cost per period of equipment. These imprecise data make the objective function also imprecise as $\tilde{z}$. They are assumed to have triangular possibility distributions, which are discussed in the next section. The equivalent uniform annual worth (EUAW) is used to represent equipment cost in each period, which means the same amount in each period.

2.2.2 Constraints

Constraints on labor levels:

$$\sum_{f=1}^{3} W_{f,t} = \sum_{f=1}^{3} W_{f,t-1} + H_t - L_t, \quad \forall t \tag{2}$$

$$W_{t\min} \le \sum_{f=1}^{3} W_{f,t} \le W_{t\max}, \quad \forall t \tag{3}$$

(2) represents labor balance. Lower and upper limit of labor capacity are demonstrated by (3).

Constraint on inventory balance:

$$I_{i,t-1} - B_{i,t-1} + P_{i,t} + O_{i,t} + S_{i,t} - I_{i,t} + B_{i,t} \cong \tilde{D}_{i,t}, \ \forall i, \ \forall t. \tag{4}$$

Where $\cong$ denotes a soft equation. In real-world, the forecast demand $\tilde{D}_{i,t}$ cannot be obtained precisely. Thus, the inventory balance equation is also imprecise as shown in (4).

Constraints on capacity and investment:

There are three types of resources for production; labor, equipment, and machine. Workers can be replaced by equipment to increase system capacity. However, additional investment is needed.

$$\sum_{i=1}^{N} a_i P_{i,t}^{w} \le \delta \times n_t \times \sum_{f=1}^{3} W_{f,t}, \quad \forall t \tag{5}$$

$$\sum_{i=1}^{N} a_i O_{i,t}^{w} \le \delta \times h_t \times \sum_{f=1}^{3} W_{f,t}, \quad \forall t \tag{6}$$

$$\sum_{i=1}^{N} b_i P_{i,t}^{e} \le 3 \times \delta \times n_t \times E_t, \quad \forall t \tag{7}$$

$$\sum_{i=1}^{N} b_i O_{i,t}^{e} \le 3 \times \delta \times h_t \times E_t, \quad \forall t \tag{8}$$

$$P_{i,t}^{w} + P_{i,t}^{e} = P_{i,t}, \quad \forall i, \ \forall t, \tag{9}$$

$$O_{i,t}^{w} + O_{i,t}^{e} = O_{i,t}, \quad \forall i, \ \forall t, \tag{10}$$

$$\sum_{i=1}^{N} c_i (P_{i,t} + O_{i,t}) \le M_t, \quad \forall t, \tag{11}$$

$$M_0 + \gamma(E_t - E_0) = M_t, \quad \forall t, \tag{12}$$

$$E_t = E_{t-1} + A_t, \quad \forall t, \tag{13}$$

$$M_t \le MC_{t\max}, \quad \forall t, \tag{14}$$

$$E_t \le E_{\max}, \quad \forall t. \tag{15}$$

Regular and overtime production by workers are represented by (5) and (6). Regular production by workers may be replaced by equipment to increase system capacity. Regular and overtime production by equipment can be illustrated by (7) and (8). Total regular and overtime production for both workers and equipment are shown in (9) and (10). Machine capacity is shown in (11). When the labor is replaced by the equipment, unloading time of the finished part and loading time of the new part are reduced so there is more remaining time for production by the machine. It means that equipment can increase machine capacity by (12). The number of equipment in each period is represented by (13). Maximum machine capacity in each period is limited as indicated in (14) and maximum number of equipment is limited by (15).

Constraint on finance:

$$\sum_{i=1}^{N} \left[\tilde{c}_{mi} \left(P_{i,t} + O_{i,t} + S_{i,t} \right) + c_{OTi} a_i O_{i,t}^{w} + c_{Ii} I_{i,t} + c_{Bi} B_{i,t} + c_{Si} S_{i,t} \right] +$$

$$c_{Wt} \sum_{s=1}^{3} W_{f,t} + c_{Ht} H_t + c_{Lt} L_t + \tilde{c}_E E_t \le \tilde{F}_{t\max}, \forall t.$$

(16)

$\tilde{F}_{t\max}$ denotes imprecise maximum available financial capacity in period *t*. All expenses in each period should not exceed this available financial capacity.

Non-negativity constraints on decision variables

$$W_{f,t}, H_t, L_t, A_t, E_t, M_t, P_{i,t}, P_{i,t}^{w}, P_{i,t}^{e},$$

$$O_{i,t}, O_{i,t}^{w}, O_{i,t}^{e}, I_{i,t}, S_{i,t}, B_{i,t} \ge 0, \forall i, \forall t. \tag{17}$$

3. Model Development

3.1 Imprecise Data with Triangular Possibility Distribution

The possibility distribution can be stated as the degree of occurrence of an event with imprecise data. Fig.1 presents the triangular possibility distributions of imprecise information. For example, in the triangular possibility distribution of $\tilde{c}_{mi} = (c^p_{mi}, c^m_{mi}, c^o_{mi})$ for all i such that $c^p_{mi} \geq c^m_{mi} \geq c^o_{mi}$. c^m_{mi} is the most possible value (possibility = 1 if normalized), c^p_{mi} (the most pessimistic value), and c^o_{mi} (the most optimistic value). Thus, the possibility distribution ($\pi_{c_{mi}}$) discussed in this study can be expressed as the degree of the occurrence of an event as being the analogous of probability distributions. DM can construct the triangular possibility distribution of $\tilde{c}_{mi}$ based on the three prominent data; the most pessimistic value, (c^p_{mi}), the most possible value (c^m_{mi}), and the most optimistic value (c^o_{mi}) of $\tilde{c}_{mi}$. By the same manner, imprecise data of $\tilde{c}_E$, $\tilde{F}_{t\max}$, $\tilde{D}_{i,t}$ and $\tilde{Z}$ can be modeled with triangular possibility distributions as follows:

$\tilde{c}_E = (c^p_E, c^m_E, c^o_E)$, such that $c^p_E \geq c^m_E \geq c^o_E$.

$\tilde{F}_{t\max} = (F^p_{t\max}, F^m_{t\max}, F^o_{t\max})\ \forall t$, such that $F^p_{t\max} \leq F^m_{t\max} \leq F^o_{t\max}$.

$\tilde{D}_{i,t} = (D^p_{i,t}, D^m_{i,t}, D^o_{i,t}), \forall i\ \forall t$, such that $D^p_{i,t} \leq D^m_{i,t} \leq D^o_{i,t}$.

$\tilde{Z} = (Z^p, Z^m, Z^o)$, such that $Z^p \leq Z^m \leq Z^o$.

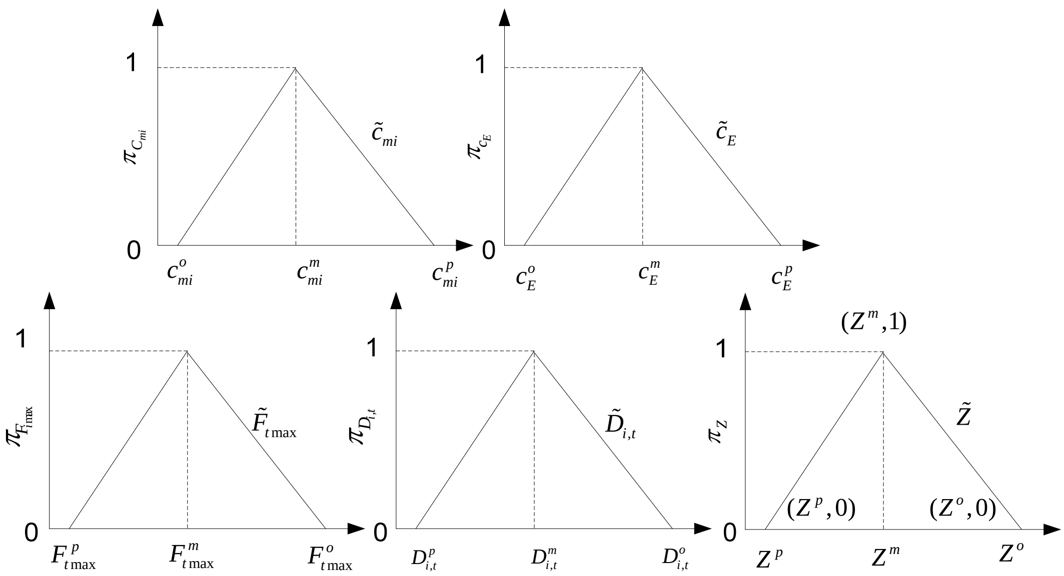

Fig. 1 Triangular possibility distributions of imprecise information of $\tilde{c}_{mi}, \tilde{c}_E$, $\tilde{F}_{t\max}, \tilde{D}_{i,t}$ and $\tilde{Z}$.

3.2 An Auxiliary Multiple Objective Linear Programming Model

3.2.1. Solving the Imprecise Objective Function

Geometrically, this imprecise objective function ($\tilde{Z}$) is fully defined by three prominent points (Z^p,0), (Z^m,1) and (Z^o,0) as shown in Fig.1. The imprecise objective can be maximized by pushing these prominent points toward the right. Because of the vertical coordinated of these points being fixed to either 1 or 0, the three horizontal coordinated are the only considerations. Consequently, solving the imprecise objective requires maximizing Z^p, Z^m, Z^o

simultaneously. This result is presented below for three new crisp objective functions.

Max Z_1 = Max Z^m. (18)

Max Z_2 = Max Z^p. (19)

Max Z_3= Max Z^o. (20)

Where:

$$Z^m = \sum_{i=1}^{N}\sum_{t=1}^{T} r_i\, D_{i,t}^m - \sum_{i=1}^{N} r_i\, B_{i,T} - \sum_{i=1}^{N}\sum_{t=1}^{T} c_{mi}^m \left(P_{i,t}+O_{i,t}+S_{i,t}\right) - [\sum_{i=1}^{N}\sum_{t=1}^{T}\left(c_{OTi} a_i O_{i,t}^w + c_{Ii} I_{i,t} + c_{Bi} B_{i,t} + c_{Si} S_{i,t}\right) + \sum_{t=1}^{T}\left(\left(\sum_{f=1}^{3} c_{Wt} W_{f,t} + c_{Ht} H_t + c_{Lt} L_t\right) + c_E^m E_t\right)]$$

$$Z^p = \sum_{i=1}^{N}\sum_{t=1}^{T} r_i\, D_{i,t}^p - \sum_{i=1}^{N} r_i\, B_{i,T} - \sum_{i=1}^{N}\sum_{t=1}^{T} c_{mi}^p \left(P_{i,t}+O_{i,t}+S_{i,t}\right) - [\sum_{i=1}^{N}\sum_{t=1}^{T}\left(c_{OTi} a_i O_{i,t}^w + c_{Ii} I_{i,t} + c_{Bi} B_{i,t} + c_{Si} S_{i,t}\right) + \sum_{t=1}^{T}\left(\left(\sum_{f=1}^{3} c_{Wt} W_{f,t} + c_{Ht} H_t + c_{Lt} L_t\right) + c_E^p E_t\right)].$$

$$Z^o = \sum_{i=1}^{N}\sum_{t=1}^{T} r_i\, D_{i,t}^o - \sum_{i=1}^{N} r_i\, B_{i,T} - \sum_{i=1}^{N}\sum_{t=1}^{T} c_{mi}^o \left(P_{i,t}+O_{i,t}+S_{i,t}\right) - [\sum_{i=1}^{N}\sum_{t=1}^{T}\left(c_{OTi} a_i O_{i,t}^w + c_{Ii} I_{i,t} + c_{Bi} B_{i,t} + c_{Si} S_{i,t}\right) + \sum_{t=1}^{T}\left(\left(\sum_{f=1}^{3} c_{Wt} W_{f,t} + c_{Ht} H_t + c_{Lt} L_t\right) + c_E^o E_t\right)]$$

Equations (18)-(20) are equivalent to simultaneously maximizing the most possible, the pessimistic, and the optimistic values of the total profit. These three objective functions are different from Lai and Hwang method [18] that uses the strategy of simultaneously maximizing the most possible value of the imprecise total profit, minimizing the risk of obtaining lower total profit and maximizing the possibility of obtaining higher total profit. The last two objective functions are relative measures from the most possible value.

3.2.2 Solving the imprecise technological coefficients and available resources

In the real-world decision problems, the DM can estimate a possibility distribution of imprecise demand based on experience and knowledge. The weighted average method may be adopted as shown in Ref. 29. However, it is difficult to assign weight for each objective. So, in the proposed method the defuzzification method is adopted to convert $\tilde{D}_{i,t}$ into a crisp number, $\tilde{D}_{i,t}$ using the following equation [30].

$$\bar{D}_{i,t} = \frac{\int D_{i,t}.\pi_{D_{i,t}} dD_{i,t}}{\int \pi_{D_{i,t}} dD_{i,t}}, \qquad \forall i, \forall t. \tag{21}$$

This is called centroid defuzzification. Then, equation (4) becomes

$$I_{i,t-1} - B_{i,t-1} + P_{i,t} + O_{i,t} + S_{i,t} - I_{i,t} + B_{i,t} = \bar{D}_{i,t}, \ \forall i, \forall t. \tag{22}$$

Moreover, in order to solve (16) with imprecise technological coefficient and available resources, the proposed approach converts these imprecise inequality constraints into a crisp one using the fuzzy ranking concept. Consequently, the auxiliary inequality (16) can be presented as follows:

$$\sum_{i=1}^{N}\left[c_{mi}^p\left(P_{i,t}+O_{i,t}+S_{i,t}\right) + c_{OTi} a_i O_{i,t}^w + c_{Ii} I_{i,t} + c_{Bi} B_{i,t} + c_{Si} S_{i,t}\right] + c_{Wt}\sum_{s=1}^{3} W_{f,t} + c_{Ht} H_t + c_{Lt} L_t + c_E^p E_t \le F_{t\max}^p, \forall t, \tag{23}$$

$$\sum_{i=1}^{N}\left[c_{mi}^m\left(P_{i,t}+O_{i,t}+S_{i,t}\right) + c_{OTi} a_i O_{i,t}^w + c_{Ii} I_{i,t} + c_{Bi} B_{i,t} + c_{Si} S_{i,t}\right] + c_{Wt}\sum_{s=1}^{3} W_{f,t} + c_{Ht} H_t + c_{Lt} L_t + c_E^m E_t \le F_{t\max}^m, \forall t, \tag{24}$$

$$\sum_{i=1}^{N}\left[c_{mi}^o\left(P_{i,t}+O_{i,t}+S_{i,t}\right) + c_{OTi} a_i O_{i,t}^w + c_{Ii} I_{i,t} + c_{Bi} B_{i,t} + c_{Si} S_{i,t}\right] + c_{Wt}\sum_{s=1}^{3} W_{f,t} + c_{Ht} H_t + c_{Lt} L_t + c_E^o E_t \le F_{t\max}^o, \quad \forall t. \tag{25}$$

3.2.3 Solving the auxiliary multiple objective linear programming problem

The auxiliary MOLP problems (Z_1, Z_2 and Z_3) can be converted into an equivalent single goal problem using fuzzy linear programming method by Zimmermann. The positive ideal solutions (PIS) and negative ideal solutions (NIS) of the three objective functions can be specified by optimizing each objective function with respect to the related constraints [30]. After maximizing Z_1, the optimal solution is Z_1^{PIS} and the solutions of Z_2 and Z_3 can also be obtained as Z_2^1 and Z_3^1, respectively. The solutions of maximizing Z_1, Z_2 or Z_3 can be found as shown in each row of Table 1. After obtaining all of single objective solutions, the minimum values of each objective are also gained (minimum values are presented in each column). These minimum values are Z_k^{NIS}, for k = 1, 2, 3. k is the number of the auxiliary objective functions.

For each objective function, the corresponding linear membership function is defined by

$$\mu_k(Z_k) = \begin{cases} 1, & Z_k > Z_k^{PIS} \\ \dfrac{Z_k - Z_k^{NIS}}{Z_k^{PIS} - Z_k^{NIS}} & Z_k^{NIS} \le Z_k \le Z_k^{PIS} \\ 0, & Z_k < Z_k^{NIS} \end{cases}, \; k = 1, 2, 3 \quad (26)$$

where k is the number of the auxiliary objective functions. Fig. 2 shows the linear membership function for (26).

Table 1 Payoff table of PIS.

	Z_1	Z_2	Z_3
Max Z_1 = Max z^m	Z_1^{PIS}	Z_2^1	Z_3^1
Max Z_2 = Max z^p	Z_1^2	Z_2^{PIS}	Z_3^2
Max Z_3 = Max z^o	Z_1^3	Z_2^3	Z_3^{PIS}
	Z_1^{NIS}	Z_2^{NIS}	Z_3^{NIS}

Note: Z_k^j is the optimal solution of Z_k when Z_j is optimized and Z_k^{NIS} is the minimum value in each column.

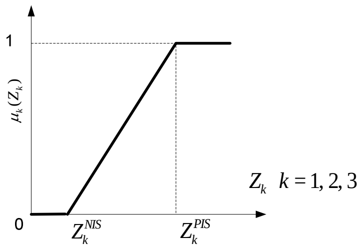

Fig. 2 Linear membership functions of Z_k.

Using the fuzzy programming method, the complete equivalent single goal model can be formulated as:

Max α,

$\alpha \le \alpha_k$, k = 1,2,3,

$\alpha_k \le \mu_k(Z_k)$, k = 1,2,3,

(2),(3),(5)-(15),(17),(22)-(25).

α is the overall satisfaction level and α_k is the satisfaction level of objective k.

Then, the optimal solution can be obtained. DM can easily modify the model interactively if the solution is not satisfied by directly modifying the degree of satisfaction of the objective functions, membership functions or parameters. Meanwhile, by Lai and Hwang method [18], DM has to manipulate the degree of satisfaction of relative measures from the most possible value objective functions (Z^m-Z^p and Z^o-Z^m) and the degree of satisfaction of the most possible value objective function. It is quite difficult to adjust the level of satisfaction of relative measures from the most possible value objective functions in order to obtain the satisfaction levels of pessimistic and optimistic objective functions (Z^p and Z^o).

4. Numerical Example

The proposed approach was applied in a manufacturing company. This company produces plastic products for both export and domestic uses. Workers are used to load and unload parts to and from machines. This company was considering replacement of workers by robot arms for shortening the loading and unloading times, improving system efficiency and competitiveness.

In this case study two product groups with an 8-period planning horizon are analyzed under the following conditions:

The regular time per worker (δ) is 8 hours. Beginning workforce level (W_0) is 90 persons. Number of workers in all shifts are equal ($W_{1,t}$ = $W_{2,t}$ = $W_{3,t}$).

The cost of regular payroll (c_{Wt}) is ฿ 15,000 per man. The cost of hiring (c_{Ht}) and layoff (c_{Lt}) are ฿ 20,000 and ฿ 45,000 per man, respectively. The cost of overtime (c_{OTi}) is ฿ 2 per unit.

Initially, there is no backorder ($B_{i,0}$ = 0) and initial inventory ($I_{i,0}$) of both products is 500 units. Subcontracting is not allowed in this case.

Equipment cost ($\tilde{c}_E$) = (฿ 150,000, ฿ 100,000, ฿ 70,000). There is no investment in equipment at the beginning period (E_0 = 0).

Increased number of machine-hour per unit of equipment in each period (γ) is 1,000 hours. The maximum number of equipment is limited to 40 units.

Available financial resource in each period ($\tilde{F}_{t\max}$) is (฿ 80 million, ฿ 100 million, ฿ 120 million).

Operating time and other cost data are presented in Table 2 and resource capacity is shown in Table 3. Fuzzy demand is presented in Table 4.

Table 2 Operating time and operating cost data.

Product	a_i (man-hr/unit)	b_i (equipment-hr/unit)	c_i (m/c-hr/unit)	$\tilde{c}_{mi}$	r_i	c_{Ii}	c_{Bi}
1	0.067	0.04	0.045	(50, 40, 35)	80	4	20
2	0.083	0.058	0.065	(85, 75, 70)	150	7.5	35

Table 3 Resource capacity data.

	Period							
	1	2	3	4	5	6	7	8
$W_{t\max}$	500	500	500	500	500	500	500	500
$W_{t\min}$	30	30	30	30	30	30	30	30
M_0	33,480	30,240	33,480	36,800	33,480	30,240	33,480	36,800
$MC_{t\max}$	6,960	60,480	66,960	73,600	66,960	60,480	66,960	73,600
n_t	60	55	62	60	60	55	62	60
h_t	30	32	28	30	30	32	28	30

Table 4 Fuzzy demand data.

	Period 1	Period 2	Period 3	Period 4
$\tilde{D}_{1,t}$	(520000, 550000, 640000)	(510000, 540000, 600000)	(900000, 930000, 960000)	(700000, 720000, 740000)
$\tilde{D}_{2,t}$	(52000, 55000, 64000)	(51000, 54000, 60000)	(90000, 93000, 96000)	(70000, 72000, 74000)
	Period 5	Period 6	Period 7	Period 8
$\tilde{D}_{1,t}$	(580000, 610000, 670000)	(600000, 610000, 620000)	(950000, 980000, 1040000)	(750000, 790000, 830000)
$\tilde{D}_{2,t}$	(58000, 61000, 67000)	(60000, 61000, 62000)	(95000, 98000, 104000)	(750000, 79000, 83000)

5. Results and Discussions

The proposed model is different from the conventional APP model in two aspects. First, the proposed model considers the labor replacement by equipment. Second, it considers imprecise data for some parameters. Therefore, there are two analysis parts in this section. In section 5.1 the benefits of the labor replacement by equipment will be analyzed. The benefits of imprecise data and the effectiveness of the proposed fuzzy linear programming method for determining the compromised solution are analyzed in section 5.2.

5.1 Benefits Obtained When the Labor Replacement by Equipment is Allowed

In order to clearly show the benefits of labor replacement by equipment, it is assumed that all parameters are precise and represented by the most likely data. As a result, there is a single objective to maximize the most likely profit. Table 5 shows the optimal solution of the proposed model where the labor replacement is allowed and that of the conventional APP model where the labor replacement is not allowed (E_t =0). It was found that the proposed model can extremely increase the profit from ฿ 1.98595x10^8 to ฿ 2.60089x10^8 by investing 6 equipment in period 2, 1 equipment in period 3, 2 equipment in period 6 and 1 equipment in period 7. When labor replacement is not allowed, workforce level is increased to increase capacity but in the proposed model equipment is invested to increase capacity and workforce level can be reduced. Equipment can perform tasks faster than labor so system capacity and efficiency can be increased.

Table 5 Optimal solutions of the proposed model with most likely data and the conventional model (E_t = 0).

	Period							
	1	2	3	4	5	6	7	8
	The proposed model: Z_1 = 2.60089x10^8							
$P_{1,t}$, $P_{2,t}$ (x100)	4,263, 0	4973, 0	4,888, 930	5,395, 0	4,970, 178	5,404, 0	5,272, 990	5,310, 790
$O_{1,t}$,$O_{2,t}$ (x100)	1,432, 565	2,212, 550	2,728, 0	1,805, 720	1,230, 443	2,389, 610	2,935, 0	2,590, 0
$I_{1,t}$, $I_{2,t}$ (x100)	0, 0	1,685, 0	0, 0	0, 0	0, 0	1,693, 0	0, 0	0, 0
A_t	0	6	1	0	0	2	1	0
$B_{1,t}$, $B_{2,t}$	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
$W_{1,t}$, $W_{2,t}$, $W_{3,t}$	20, 20, 20	16, 16, 16	16, 16, 16	14, 14, 14	13, 13, 13	13, 13, 13	13, 13, 13	13, 13, 13
M_t	33,480	35,907	40,313	43,633	40,313	39,034	43,366	46,686
	The conventional model (E_t =0): Z_1 =1.98595x10^8							
$P_{1,t}$, $P_{2,t}$ (x100)	5,353, 0	4,226, 550	5,532, 0	4,461, 720	5,344, 0	4,898, 0	5,522, 0	4,365, 790
$O_{1,t}$,$O_{2,t}$ (x100)	1,271, 565	1,700, 0	565, 930	2,677, 0	1,201, 620	941, 610	488, 990	2,672, 0
$I_{1,t}$, $I_{2,t}$ (x100)	929, 0	1,354, 0	0, 0	0, 0	0,0	0,0	0, 0	0, 0
A_t	0	0	0	0	0	0	0	0
$B_{1,t}$, $B_{2,t}$	0, 0	0, 0	1,849, 0	1,911, 0	1,567, 0	1,828, 0	5,718, 0	6,581, 0
$W_{1,t}$, $W_{2,t}$, $W_{3,t}$	25, 25, 25	25, 25, 25	25, 25, 25	25, 25, 25	25, 25, 25	25, 25, 25	25, 25, 25	25, 25, 25
M_t	33,480	30,240	33,480	36,800	33,480	30,240	33,480	36,800

Table 6 Efficient compromised plan of the proposed model.

Initial compromised solution from the proposed PLP model α = 0.787444, α_1= 0.928409, α_2= 0.787444, α_3= 0.787444, Z_1 = 2.60060x10^8 , Z_2 =1.67332x10^8, Z_3 = 3.19979x10^8, PIS = (260,089,000, 167,707,000, 320,201,000), NIS = (259,678,000, 165,943,000, 319,157,000)								
Lower risk compromised solution α = 0.664176, α_1= 0.664176, α_2= 0.85, α_3= 0.664176, Z_1 = 2.60021x10^8 , Z_2 =1.67442x10^8, Z_3 = 3.19850x10^8, PIS = (260,089,000, 167,707,000, 320,201,000), NIS = (259,678,000, 165,943,000, 319,157,000)								
	Period							
	1	2	3	4	5	6	7	8
$P_{1,t}$, $P_{2,t}$ (x100)	4,263, 0	4,559, 550	5,961, 0	5,395, 0	5,395, 0	5,140, 0	6,218, 0	6,017, 0
$O_{1,t}$,$O_{2,t}$ (x100)	1,432, 565	2,739, 0	1,540, 930	1,805, 720	1,631, 620	2,235, 610	1,582, 990	1,883, 790
$I_{1,t}$, $I_{2,t}$ (x100)	0, 0	1,798, 0	0, 0	0, 0	825, 0	2,101, 0	0, 0	0, 0
A_t	0	7	0	0	0	0	1	0
$B_{1,t}$, $B_{2,t}$	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
$W_{1,t}$, $W_{2,t}$, $W_{3,t}$	20, 20, 20	17, 17, 17	16, 16, 16	15, 15, 15	15, 15, 15	15, 15, 15	15, 15, 15	15, 15, 15
M_t	33,480	36,418	39,803	43,123	39,803	37,154	41,532	44,852

5.2 Advantages of the PLP APP Model over the Conventional APP Model

The conventional APP model assumes that all parameters are precise and represented by the most likely data. There is no information of the optimistic and pessimistic data. Thus, it is only possible to maximize the most likely profit. In this case the optimal most likely profit is ฿2.60089x10^8 (the proposed model in Table 5). In contrast, the proposed PLP model provides three values of the profit under most likely, pessimistic, and optimistic scenarios of the compromised solution. The three values of the profits are ฿1.67332x10^8 (pessimistic value, Z_2), ฿2.60060x10^8 (most likely value, Z_1), and ฿ 3.19979x10^8 (optimistic value, Z_3) as shown by the initial compromised solution in Table 6. The solution from the proposed approach supplies more useful information to the decision maker. They are well informed of the pessimistic and optimistic cases as well.

Moreover, the proposed PLP model generates a compromised solution by simultaneously maximizing the most likely, pessimistic, and optimistic values of the imprecise total profit. The overall satisfaction level of the initial compromised solution is 0.787444 as show in Table 6. The satisfaction levels for the most likely, pessimistic, and optimistic profits are 0.928409, 0.787444, and 0.787444, respectively. This means that the obtained compromised solution has a higher degree of satisfaction for the most likely profit than that for the pessimistic and optimistic profits.

Finally, DMs can manipulate the compromised solution based on their preferences. Suppose the decision makers prefer a lower risk solution (higher pessimistic profit), they can easily manipulate α_2 and solve for the new compromised solution. If the constraint that $\alpha_2 \geq$ 0.85 (previously α_2 = 0.787444) is added to the PLP model, the production plan of lower risk compromised solution is obtained as shown in Table 6. This is easier to manipulate than using Lai and Hwang Method. The production plan of lower risk compromised solution has higher pessimistic profit but lower most likely and optimistic profits than the compromised solution without any condition. Some decision variables of the lower risk compromised solution are also presented in Table 6. It is observed that the production plan of lower risk compromised solution in Table 6 requires less investment in equipment than the solution of the proposed model in Table 5, which is a noncompromised solution that only maximizes the most likely profit.

6. Conclusions

This work presents a Possibilistic Linear Programming (PLP) approach for solving the multi-product Aggregate Production Planning (APP) problem with labor replacement in a parallel machine environment where some information are imprecise. The proposed approach attempts to maximize the total profit. It uses the strategy of simultaneously maximizing the most possible, the pessimistic, and the optimistic values of the imprecise total profit. The proposed model yields an optimal compromised APP solution. Labor replacement by equipment can highly increase profit and capacity of the system. The proposed PLP model is appropriate for practical use because it can handle imprecise data. The DM can easily interactively adjust the satisfaction levels of each objective function by changing the degree of satisfaction of objective functions, membership functions, or parameters until a satisfactory solution is reached. The production plan of the compromised solution can reduce the risk of getting too low profit in pessimistic case. Thus, it is more preferable for decision-making.

This paper has three major contributions as follows. First, this paper develops the APP model that explicitly considers labor replacement by equipment and demonstrates that the labor replacement can significantly increase the total profit of the company. Second, it shows how imprecise data are handled by formulating the PLP model with three explicit objectives of maximizing pessimistic, most likely, and optimistic profits. This PLP is easier to manipulate than the existing PLP model. Finally, this paper applies the fuzzy programming method to determine an initial compromised solution and a lower risk compromised solution based on preference of the decision maker. The production plan with low risk may be more preferable than the higher total profit one with high risk. Methodologies presented in this paper have significant values for practitioners who want to apply APP model in realistic situations.

The research in APP area may be extended to incorporate a marketing promotional plan. An example of research questions includes what type of marketing promotions should be offered in what period (peak or off-peak) to maximize the imprecise profit due to imprecise information

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